Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Student number\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

*L*

1

2

*g*

*z,Z*

*x,X*

1

**Assignment** **4**

A beam is loaded by its own weight as shown in the figure. Assume that displacement is confined to the plane. Derive the equilibrium equations for buckling analysis giving the axial displacement and the critical density  of the material. Start with the virtual work density and approximations to the axial and transverse displacements. The cross-section properties *A, I* and material properties *E,*  are constants.

**Solution template**

Virtual work expressions for the buckling analysis of a beam in plane consist of the internal parts for the bar and bending modes, coupling (stability expression) between them, and virtual work of the external point force. Altogether ()

, where .

In terms of the non-zero displacement/rotation components of the structural system, approximations to the axial displacement *u,* transverse displacement *w*, and the axial force *N* simplify to

,

,

.

When the approximations are substituted there, virtual work density simplifies to (substitute the expression for the axial forceand distributed force)





Integration over the length of the beam gives



 

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Principle of virtual work and the fundamental lemma of variation calculus imply equilibrium equations

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The first equation is linear and can be solved for the axial displacement

  .

When the solution to the axial displacement is substituted there, the second (non-linear) equation simplifies to

.

The remaining task is to deduce the possible solutions: If the expression in parenthesis is non-zero, the equation implies that . If the expression in parenthesis is zero, the equation is satisfied no matter the non-zero value of . Therefore, buckling may occur when (here density  stands for the loading parameter)

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